Single Pure – Factorising Hard

Patrons are reminded that you should always pull out the *lowest* powers from an expression. For example

$$\begin{aligned}
2x^3\sqrt{x-1} - \frac{4x^4}{\sqrt{x-1}} &= 2x^3(x-1)^{\frac{1}{2}} - 4x^4(x-1)^{-\frac{1}{2}} \\
&= 2x^3(x-1)^{-\frac{1}{2}}\big[(x-1) - 2x\big] \\
&= 2x^3(x-1)^{-\frac{1}{2}}(-x-1) \\
&= -2x^3(x-1)^{-\frac{1}{2}}(x+1) \\
&= -\frac{2x^3(x+1)}{\sqrt{x-1}}.
\end{aligned}$$

Now factorise the following expressions fully.

1. $25x^2(2x+1) - 15x(2x+1)^2$. $\boxed{-5x(2x+1)(x+3)}$
2. $4x^2(x-3)^2 - 8x(x-3)$. $\boxed{4x(x-3)(x^2-3x-2)}$
3. $10x^3(2x-1) + 5x^2(2x-1)^2$. $\boxed{5x^2(2x-1)(4x-1)}$
4. $y(y-4)^{10} - 2y^2(y-4)^9$. $\boxed{-y(y+4)(y-4)^9}$
5. $(x+1)^4(x+3) - 2(x+1)^3(x+3)^2$. $\boxed{-(x+5)(x+3)(x+1)^3}$
6. $9a(x+1)^4(x-2)^5 - 15a(x+1)^5(x-2)^4$. $\boxed{-3a(2x+11)(x+1)^4(x-2)^4}$
7. $\dfrac{6(x+1)^3}{(2x+1)^5} - \dfrac{4(x+1)^4}{(2x+1)^6}$. $\boxed{\frac{2(4x+1)(x+1)^3}{(2x+1)^6}}$
8. $(x+1)\sqrt{x+1} - 2\sqrt{x+1}$. $\boxed{(x-1)\sqrt{x+1}}$
9. $\dfrac{(x+1)^2}{\sqrt{x-1}} - (x+1)\sqrt{x-1}$. $\boxed{\frac{2(x+1)}{\sqrt{x-1}}}$
10. $\dfrac{10\sqrt{3x+1}}{(2x-1)^4} - \dfrac{20}{(2x-1)^3\sqrt{3x+1}}$. $\boxed{\frac{10(3-x)}{(2x-1)^4\sqrt{3x+1}}}$
11. $\dfrac{(x+1)^n}{(x-1)^m} + \dfrac{(x+1)^{n+1}}{(x-1)^{m+1}}$. $\boxed{\frac{2x(x+1)^n}{(x-1)^{m+1}}}$
12. $ax^3(x+1)^7(x+2)^8 - a^2x^2(x+1)^7(x+2)^7$. $\boxed{ax^2(x^2+2x-a)(x+1)^7(x+2)^7}$
13. $\dfrac{2x+1}{\sqrt{x-1}} - (2x+1)^3\sqrt{x-1}$. $\boxed{-\frac{(2x+1)(4x^3-3x-2)}{\sqrt{x-1}}}$
14. $2k(x-2)^4(2x+1)^{\frac{4}{3}} - k\sqrt[3]{2x+1}(x-2)^5$. $\boxed{k(3x+4)(x-2)^4\sqrt[3]{2x+1}}$